A Social Choice Theoretic Comparison of Ranked Choice and First Past the Post Voting

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1 Introduction

Recently, electoral and democratic process reforms have come to the forefront of state and national political debates. At the state level, there has been a surge in the number of states passing redistricting reform, either through legislation or via ballot initiative. There has also been a grassroots push to replace first-past-the-post (FPTP), or plurality, voting with a different electoral system. One of the leading contenders is ranked choice voting (RCV).

Multiple candidates for the 2020 Democratic nomination have signalled their support for electoral reform. Pete Buttigieg has endorsed a switch to ranked choice voting as a part of his campaign that emphasizes structural reforms to America's political system.[1]

FairVote, a nonprofit organization that advocates for electoral reform, is one of the leading proponents of ranked choice voting. The group catalogues the growing list of localities that use RCV, as well as keeping track of the progress of RCV legislation in state legislatures. Legislation enabling RCV at the municipal level passed earlier this year in Utah and New Mexico, and there are currently 13 states with ranked choice voting bills introduced to the legislature.[2]

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[1] https://www.ozy.com/politics-and-power/will-the-live-free-or-die-state-usher-in-ranked-choice-voting/92951

[2] https://www.fairvote.org/new_ranked_choice_voting_in_states

In this essay I will examine the arguments made by RCV advocates and evaluate them from a social choice theoretic perspective. To keep my analysis relevant to the U.S. context, I will restrict my focus to single-member districts; this framework is relevant to essentially all federal elections in the United States, including House, Senate, and state-level presidential elections, as well as most elections at the state and local level.

2 What Is RCV

Ranked choice voting is what McGann (2006) calls a seat allocation rule: a procedure used to allocate seats in a legislature to candidates. RCV is a ranked, or positional (Riker, 1982), voting method. Unlike FPTP, where voters only cast a vote for their most-preferred candidate, in RCV voters submit an ordered preference list of the candidates. It is analogous to single transferable vote in the case of single-member districts.

RCV is also known as instant runoff voting (IRV), and as Alternative Vote in Australia, where it has been used for federal parliamentary elections for a century.

2.1 Denfining the RCV Algorithm

Let $N = 1, 2, \ldots, n$ be the set of voters, and A the finite set of alternatives. Let the preferences of each $i \in N$ be a strict total order on A. Say xP_iy if voter i prefers alternative x to alternative y. Say that a voter i first-ranks an alternative a if $aP_ib \ \forall b \neq a \in A$. For tie-breaking purposes, let there be a lexicographic ordering on the elements of $A: \ a \succ b \succ c \ldots$.

The ranked choice voting algorithm proceeds as follows. If there exists an alternative a in A such that for all$a, b \in A$,

$$|\{i : aP_ib \ \forall b \neq a \in A\}| > \frac{n}{2}$$

then a is the ranked choice winner. If not, then find the alternative $b \in A$

such that

$$\arg\min_{b \in A} |\{i : bP_i a \ \forall b \neq a \in A\}|$$

If two or more alternatives are tied for the least number of first-ranks, then select the $b \in A$ such that b is the maximal of the tied elements according to the lexicographic ordering of the alternatives.

Now let $A' = A \setminus \{b\}$. Then, repeat the first step and check if there is now an alternative that is first-ranked by a majority of voters. If not, continue iteratively eliminating alternatives as above until an RCV winner is found.

2.2 RCV in Action: Maine's 2nd Congressional District

Ranked choice voting was implemented in Maine for federal elections in 2018. In Maine's 2nd Congressional District, there was no outright majority winner, so the RCV algorithm was used to determine the winner.

Table 1: ME-2 Election Results[3]

Candidate	Round 1 Votes	Round 2 Votes
Jared Golden (D)	132,013	142,440
Bruce Poliquin (R)	134,184	138,931
Tiffany Bond (I)	16,552	0
Will Hoar (I)	6,875	0

Despite the fact that Poliquin was the plurality winnner in the first round, Golden received sufficiently many second place votes from the two independent candidates that were eliminated after the first round such that he won in the second round. This is an example, then, where the plurality and RCV outcomes differ. However, if this election was run under FPTP rules, it is very possible that the supporters of the independent candidates would have voted strategically and the outcome would have been the same.

[3] Election Results from Ballotpedia:
https://ballotpedia.org/Maine%27s_2nd_Congressional_District_election,_2018

3 What Arguments do RCV Advocates Make?

Advocates of RCV make a variety of arguments for the superiority of RCV over plurality voting methods. In this section I will present a summary of the main arguments presented by RCV advocates. In the next, I summarize the social choice theoretic literature on ranked choice voting. I then lay out some of the social choice theoretic properties of RCV that underpin these arguments.

Many of the arguments in favor of RCV address the incentives different electoral systems create for candidates to campaign in certain ways. Specifically, proponents claim that it reduces incentives for candidates to engage in negative campaigning because they want the second-place votes of their opponents' supporters. RCV advocates also argue that it is superior to top-two runoff systems (another popular modification to the FPTP system) because it does not require costly followup elections that inevitably have lower voter turnout than the first round. These are certainly relevant considerations when selecting a voting procedure, but they are outside the scope of this paper. However, social choice theory is able to speak to a number of the arguments that RCV advocates make on the system's behalf.

3.1 Electoral Competition and Choice

Proponents of ranked choice voting argue that it is a solution to the problem posed by Duverger's Law, which posits that plurality voting and single member districts promote a two-party political system and inhibit the development of effective third parties. RCV advocates argue that a two party system in a country as large and diverse as the United States is insufficient to represent the full scope of ideological diversity in the U.S.

3.2 Strategic Voting

Supporters of ranked choice voting argue that it minimizes the incentives for voters to behave strategically (i.e. submit something different than their true preference ranking) compared to pluarlity rule. FairVote claims that "with ranked choice voting, you can honestly rank candidates in order of choice without having to worry about how others will vote and who is more or less likely to win."[4] The intuition behind this argument is that a "wasted" vote for a more-preferred, but generally less supported candidate will simply be reallocated after a candidate has been eliminated.

3.3 Majority Support

Proponents of ranked choice voting argue that the RCV winner will have some notion of majority support. If there is a candidate that is first-ranked by an outright majority, that candidate will win, just as in FPTP. However, if there is no candidate ranked first by an absolute majority, RCV ensures that the eventual winner is majority-preferred to at least *some* of the other alternatives. This is not the case for FPTP, which can elect candidates that are not majority-preferred to *any* alternative. RCV advocates typically frame this argument in terms of "moderate" and "extremist" candidates.[5]

4 Literature Review

In this section, I briefly review the social choice theory literature on ranked choice voting.

Fishburn and Brams (1983) use a stylized example of a small-town election to highlight some of the paradoxes of ranked choice voting. Their broader point is that electoral reformers should be wary; a new voting system that addresses one flaw will introduce another. There is no ideal, flawless system, so any choice will necessarily have to weigh the flaws of one system against another.

Grofman and Feld (2004) compare ranked choice voting to a similar voting system, the Coombs rule, in a context where voters have single-peaked preferences along a unidimensional policy space. The authors evaluate the two systems on four criteria: Condorcet winner, Condorcet loser, resistance to strategic manipulability, and simplicity.

[4] `https://www.fairvote.org/rcv`

[5] See Grofman and Feld (2004, p. 648)

Saari (1990) uses the concept of a voting "dictionary", a collection of all possible preference profiles over a set of alternatives and all possible outcomes, to compare different positional voting methods, of which RCV and FPTP are examples. Saari asserts that "the Borda method is the unique positional voting method to minimize the kinds and number of paradoxes that can occur."[6] That is, he claims that if Borda exhibits a paradox, all other positional methods will also exhibit that paradox.

However, Saari's claim appears to be too strong. It is possible for Borda to fail to select an alternative that is first-ranked by an absolute majority of voters, while both RCV and FPTP will always select an absolute majority winner. This is clearly a paradox by Saari's definition, but RCV and plurality rule, two positional methods, do not exhibit this flaw.

The unique contribution of my paper is to compare ranked choice voting and first-past-the-post on a social choice theoretic basis. There is a great deal of academic work analyzing each of these systems in isolation, but relatively little that compares RCV as a proposed alternative electoral system to the status quo of plurality rule.

5 Social Choice Theoretic Properties of RCV

In this section, I list and prove some of the social choice theoretic properties of ranked choice voting.

5.1 Condorcet Winner

Ranked choice voting *fails* to elect the Condorcet winner in certain circumstances. Take the following example with $N = 1, \ldots, 100$ and $A = \{a, b, c\}$.

$(n = 35)$	$(n = 34)$	$(n = 31)$
a	c	b
b	b	c
c	a	a

[6] Saari (1990, p. 280)

In this example, candidate B is the Condorcet winner. However, ranked choice voting selects a different candidate. $B \succ A$ 65-35 and $B \succ C$ 66-34. However, B receives the least amount of first place votes in the first round and is consequently eliminated. In the second round, C beats A 65-35.

A voting rule's *Condorcet efficiency* is the probability that it selects the Condorcet winner, given that it exists. Grofman and Feld (2004) summarize the literature on the Condorcet efficiency of RCV and plurality rule and conclude that, at least under the assumption of Euclidean preferences, RCV has a higher Condorcet efficiency than FPTP.

The failure to elect the Condorcet winner has also happened in practice. In the 2009 Burlington, Vermont mayoral election, the RCV winner was different than the plurality winner, which in turn was different from the Condorcet winner.[7] The controversy that followed this election led Burlington to vote to repeal RCV the following year.

5.2 Condorcet Loser

Ranked choice voting never selects the Condorcet loser, if it exists. The Condorcet loser is defined analogously to the Condorcet winner: a candidate (or alternative) is a Condorcet loser if all other candidates are majority-preferred to that candidate in pairwise comparisons.

Let N be the set of n voters and X the set of alternatives. Let each $i \in N$ have a strict total order P_i on the alternatives X. Then $x \in X$ is a Condorcet loser if:

$$|\{i \in N : yP_ix\}| > |\{i \in N : xP_iy\}|, \ \forall y \neq x \in X$$

Claim 1. *Ranked choice voting never selects a Condorcet loser.*[8]

Proof. Let F, the social decision function (or seat allocation rule) be ranked choice voting as defined above in Section 2.1. Suppose there exists some

[7] See Ornstein and Norman (2014) for a more detailed analysis of this election.

[8] This claim cannot be strengthened to say that a Condorcet loser will always be eliminated in the first round. Consider the case with 5 voters and 3 candidates, where 3 voters rank $a \succ c \succ b$ and 2 rank $b \succ c \succ a$. In this example, b is a Condorcet loser, but c is eliminated first.

alternative $z \in X$ that is a Condorcet loser. Because z is a Condorcet loser, there exists no subset of X in which z garners a majority of first place votes. By the definition of the Condorcet loser,

$$|\{i \in N : zP_i x\} < \frac{1}{2}, \ \forall x \neq z \in X$$

Therefore, z cannot be chosen as the winner of a ranked choice voting process, which requires that the winning outcome achieves an absolute majority of first-place votes against some subset of X. □

5.3 Monotonicity

Fishburn and Brams (1983) show that ranked choice voting fails the monotonicity criterion (what they call the "more-is-less" paradox). Weak monotonicity is equivalent to non-negative responsiveness as defined in McGann (2006). McGann defines monotonicity as "the requirement that if some voters switch to an alternative and everything else remains equal, it cannot do worse than before."[9]. Table 2 shows an example with 3 alternatives and 17 voters where RCV fails the monotonicity criterion.

Table 2: RCV Violates Monotonicity

$(n = 3)$	$(n = 2)$	$(n = 4)$	$(n = 2)$	$(n = 4)$	$(n = 2)$
a	a	b	b	c	c
b	c	a	c	a	b
c	b	c	a	b	a

With the preferences listed above, a is eliminated in the first round, as it has fewer first place votes (5) than b or c (6). After the votes have been re-allocated, b beats c by a votes of 9-8.[10]

Now suppose that the two voters who have the preference ordering $c \succ b \succ a$ instead voted $b \succ c \succ a$, increasing the support for b. Now, c is eliminated after the first round, and a will beat b in the second round.

[9]McGann (2006, p. 18)

[10]Also note that this is another example where RCV fails the Condorcet winner criterion. a is majority-preferred to both b and c

Ornstein and Norman (2014) conduct simulations of a spatial model of voting in a two-dimensional policy space with three candidates. Voters have Euclidean preferences and candidates adapt their policy position (a point in the policy space) to a series of pre-election polls. Ornstein and Norman find that, in competitve elections (where each of the three candidates get at least 25% of the vote), monotonicity failure occurs at least 15% of the time, depending on which stylized distribution of voters they used.

The Burlington, VT mayoral election described above in Section 5.1 also exhibits a violation of monotonicity.

5.4 Nash Implementability

Theorem 2 from Maskin (1999) shows that monotonicity is a necessary condition for a social choice rule to be implementable in Nash equilibrium.

Theorem 2. *(Maskin (1999)) If $f \twoheadrightarrow A$ is an SCR that is implementable in Nash equilibrium, then it is monotonic.*

RCV fails monotonicity. Therefore, it follows directly from the contrapositive of Theorem 2 that RCV is not implementable in Nash equilibrium.

5.5 IIA: Contraction Consistency

The preference profile presented in Table 2 also demonstrates another property that RCV fails to satisfy: the contraction consistency version of independence of irrelevant alternatives (what Mackie (2003) calls IIA-RM).

Let A be the set of alternatives and $C(\cdot)$ be the choice set correspondence for some social choice rule. Contraction consistency requires that

$$x \in C(A) \wedge x \in A' \subset A \Rightarrow x \in C(A')$$

In other words, an alternative chosen from a larger set must also be chosen from a subset of that larger set when it is available.

As discussed above, applying RCV the preference profiles in Table 2 yields B as the social choice. However, if we strike C from the preference profiles, we get the following reduced preference lists:

$(n = 9)$	$(n = 8)$
A	B
B	A

This direct comparison yields A as the winner, despite the fact that B is available. Therefore, RCV violates contraction consistency.

5.6 Truncation

The truncation criterion requires that a voter cannot obtain a more preferable outcome (i.e. a candidate ranked higher in their personal preferences) by submitting a truncated version of their true preference list. Ranked choice voting fails the truncation criterion.

Consider the following example from Nurmi (1999), with $N = 1, \ldots, 103$ and $A = a, b, c, d$.

$(n = 33)$	$(n = 29)$	$(n = 24)$	$(n = 17)$
a	b	c	d
b	a	b	c
c	c	a	b
d	d	d	a

Under the full preference lists, d and b are successively eliminated before a wins. However, consider the following alteration to the above election, where the voters who prefer $d \succ c \succ b \succ a$ only report d.

$(n = 33)$	$(n = 29)$	$(n = 24)$	$(n = 17)$
a	b	c	d
b	a	b	
c	c	a	
d	d	d	

In this election, after d is eliminated in the first round the 17 voters are removed after their ballots have been exhausted. Next, c is eliminated (instead of b in the first election), leaving b the winner. This result is preferred to a by the 17 voters who truncated their preferences.

5.7 Later-No-Harm

The later-no-harm criterion requires that "adding a later preference to a ballot should not harm any candidate already listed." RCV's compliance with this criterion follows from the construction of the algorithm. RCV only considers votes for candidates that are *currently* top-ranked, so any additional votes below a candidate only become relevant after that candidate has been eliminated. Therefore, adding more alternatives lower down the ballot cannot harm a candidate's chances of winning.

6 Comparison of RCV and FPTP

6.1 Electoral Competition

Empirical evidence on ranked choice voting's ability to promote multiparty competition in single member districts. Australia has used RCV for federal elections since 1919, but it has a de-facto two party system. At the same time, Duverger's law isn't determinate: the U.S. and the U.K. both use FPTP and single-member districts, but the U.K. has a multiparty system while the U.S. has an entrenched two-party system. It is quite possible that the key factor is the type of system itself (i.e. presidential or parliamentary) is more important than the voting rule that is used.

Nevertheless, it is at least plausible to argue that ranked choice voting presents lower barriers to the viability of additional parties than plurality rule. The advantages of RCV when it comes to electoral competition seem more clear at the municipal level, where elections are often nonpartisan and dominated by local concerns rather than polarized national politics.

6.2 Majority Support: the Condorcet Criteria

Ranked choice voting fails the Condorcet winner criterion but passes the Condorcet loser criterion. FPTP on the other hand fails *both* of these criteria. Along this dimension then, RCV dominates FPTP as a voting mechanism. The ability of FPTP to select a Condorcet loser is a major flaw;

no reasonable definition of a democratic electoral process should admit the possibility of choosing an alternative that is beaten by every other alternative in a head-to-head comparison.

In this sense, supporters' arguments that ranked choice voting selects candidates with majority support is correct. The choice will always be a candidate with majority support against *some* subset of alternatives, if not the entire set.

6.3 Strategic Voting

On their face, the less-than-careful claims that ranked choice voting allows voters to "honestly rank candidates in order of choice without having to worry about how others will vote and who is more or less likely to win"[11] are clearly false. The example in Section 5.6 where a subset of voters obtain an outcome they prefer by truncating their preference list is clearly an example of strategic voting by any definition. The example of monotonicity failure in Section 5.3 also underscores this point.

RCV's vulnerability to strategic voting is guaranteed by the Gibbard-Satterthwaite theorem (Gibbard, 1973; Satterthwaite, 1975), which states that any non-dictatorial voting rule that admits more than two alternatives is susceptible to strategic voting. As a result, RCV advocates' claims in this domain are incorrect and misleading. Another way to understand it's guaranteed vulnerability to strategic voting follows from the Nash implementability result. If RCV cannot be implemented in *any* Nash equilibrium, then it follows that it cannot be implemented in an equilibrium where truth-telling is a dominant strategy for the voters.

In some sense, the increased size of the strategy space created by allowing voters to submit lists of preferences instead of single votes creates more, and more complex, opportunities for strategic voting compared to the FPTP setting. Additionally, the strategic incentives of a plurality system are more straightforward and predictable: FPTP elections in single member districts tend to promote two-party competition, narrowing and simplifying the viable

[11] `https://www.fairvote.org/rcv`

choice set for voters. For ranked choice voting on the other hand, the strategic incentives are much more varied; in some instances, such as Section 5.3, voters have an incentive to submit a permutation of their preference list, while in others such as the example in Section 5.6, they have incentives to truncate their true preference list.

6.4 Later-No-Harm and Truncation

Ranked choice voting passes the later-no-harm criterion but fails the tuncation criterion. FPTP satisfies both later-no-harm and the truncation criterion trivially, as it only considers one vote from each voter.

At first glance, there appears to be a contradiction in the fact that RCV satisfies one of these criteria but not the other. The distinction between these two is subtle. Later-no-harm requires that *candidates'* electoral chances cannot be harmed by including less-preferred candidates lower down on the list. Truncation instead requires that voters cannot attain an outcome that *they themselves* prefer better by truncating their submitted preference list.

Concretely, a voter can rank more candidates after their most-preferred without impacting that candidate's chance of winning, but still make the final outcome worse from their point of view. If one of the goals of choosing an electoral system is to minimize incentives for strategic behavior, I argue that the later-no-harm criterion is essentially meaningless. Voters seek to maximize the outcome according to their individual preference orderings, so the fact that it is "safe" for the candidate if a voter adds more lower-ranked choices is overridden by the fact that voters can sometimes improve their outcome by truncating their preference list.

Satisfying the later-no-harm criterion does not make an electoral rule immune from incentives to "bullet vote," or submit a ballot with only the voter's most-preferred candidate on it. In practice, bullet voting appears to be relatively common. In the 2009 Burlington, Vermont mayoral election, 2,312 of the 8,833 (26.2%) of ballots cast were for only a single candidate.

7 Conclusion

We know from Arrow's theorem, as well as the Gibbard-Satterthwaite theorem, that there is no perfectly well-behaved, incentive-compatible voting rule.

As a result, choosing a voting rule is an exercise in tradeoffs. Addressing a failure in one domain will necessarily introduce new paradoxes in another. Weighing ranked choice voting against first-past-the-post, the most impactful tradeoff is between the Condorcet loser criterion, which RCV satisfies and FPTP fails, and monotonicity, which FPTP satisfies and RCV fails.

Although both are highly undesirable, I contend that the Condorcet loser property is more fundamental. Selecting an alternative that loses a head-to-head comparison to every other is profoundly undemocratic. Additionally, the election of a Condorcet loser is the direct result of the submitted preferences. In contrast, monotonicity failure is a contingent opportunity for strategic voting presented by a profile of preferences, given that everyone else keeps the same actions.

On balance, ranked choice voting seems to be a marginal improvement over plurality rule. Although its impact would likely be limited in highly polarized national politics, it would likely do a better job than plurality rule at navigating local elections with many candidates. One recent example where RCV would plausibly outperform FPTP is the 2019 Chicago mayoral election, where 14 candidates competed in the city's nonpartisan election. In the first round of the two-round runoff election, the first and second place finishers garnered a combined 1/3 of the vote.[12] RCV is likely to outperform FPTP in elections like Chicago's, which featured a large field of alternatives with no consensus candidate.

[12] https://ballotpedia.org/Mayoral_election_in_Chicago,_Illinois_(2019)

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